AD 733923

FTD-MT-24-995-71

FOREIGN TECHNOLOGY DIVISION

GAS-DYNAMIC FUNCTIONS OF A POINT EXPLOSION

by

V. P. Korobeynikov, P. I. Chushkin
and K. V. Sharovatova

Approved for public release;
distribution unlimited.

57

FTD-MT-24-995-71

EDITED MACHINE TRANSLATION

GAS-DYNAMIC FUNCTIONS OF A POINT EXPLOSION

By: V. P. Korobeynikov, P. I. Chushkin and K. V. Sharovatova

English pages: 49

Source: Gazodinamicheskiye Funktsii Tochechnogo Vzryva, 1969, Moscow, AN SSSR Vychislitel'nyy Tsentr, pp. 1-48.

This document is a SYSTRAN machine aided translation, post-edited for technical accuracy by: SSgt René Courville

UR/0000/69/000/000

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gas Dynamics						
Adiabatic Process						
Adiabatic Flow						
Heat of Explosion						
Hypersonic Aerodynamics						

UNCLASSIFIED
Security Classification

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Technology Division Air Force Systems Command U. S. Air Force	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

GAS-DYNAMIC FUNCTIONS OF A POINT EXPLOSION

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Translation

5. AUTHOR(S) (First name, middle initial, last name)

Korobeynikov, V. P.; Chushkin, P. I. and Sharovatova, K. V.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1969	49	9

8a. CONTRACT OR GRANT NO. b. PROJECT NO. AARY c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) FTD-MT-24-995-71 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Foreign Technology Division Wright-Patterson AFB, Ohio

13. ABSTRACT

This article is dedicated to the problem of a point explosion with counterpressure in a quiescent nonviscous and nonheat-conducting ideal gas with the constant adiabatic given exponent. Given pressure and density of the undisturbed gas are considered constants. As a result of numerical solution of this problem, tables of the gas-dynamic functions of a point explosion are calculated. This article presents tables for the flat, cylindrical, and spherical explosions in an ideal gas for the given values equal to 1.3, 1.4, and 5/3. The tables contain the basic functions describing the field of the gas flow over a wide time span. Also, some possible applications of these tables for solving other problems in the gas-dynamics of explosion and in hypersonic aerodynamics are explained in the introductory section. The literature, already contains tables for an explosion in a gas with counterpressure. However, these tables are only for the given spherical case. Let us note also that we first calculated the tables of the gas-dynamic functions for the initial stage of a point explosion, i.e., for sufficiently large pressure gradients at the shock wave front. The tables now being published are the natural continuation of the tables for the initial stage of an explosion.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

Responsible Editor Candidate of Physics and Mathematics V. P. Karlikov

Translator's note: On several occasions, symbols found in formulae and calculations appear to have been rendered incorrectly in the original document. They will be shown exactly as they appear in the original.

TABLE OF CONTENTS

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

INTRODUCTION

The problem concerning a point explosion in a gas consists, as is known [1, 2], in determining the parameters of the gas flow, which has been caused by the instantaneous release of finite energy E° at a point (spherical explosion), along a straight line (cylindrical explosion) or along a plane (flat explosion). In the cylindrical case the energy of the explosion E° is calculated per unit length, and in the flat case – per unit area.

The problem concerning a point explosion is one of the basic transitional problems of the mechanics of a continuous medium. It can be formulated and examined both for various ideal compressible mediums and also for continuous media, in which it is impossible to disregard viscous stresses. When studying this problem for a gas, in a number of cases one should take into account influence of various physical and chemical parameters and phenomena on the flow. Thus, it is possible to take into consideration the variability of the initial density of the gas, to investigate the effect of electromagnetic fields on the motion of an electroconductive gas, to calculate the chemical reactions during an explosion in a combustible mixture, and to examine the processes of equilibrium and nonequilibrium dissociation and ionization. The questions mentioned above in the point explosion theory are now being developed by various authors.

This article is dedicated to the problem of a point explosion with counterpressure in a quiescent nonviscous and nonheat-conducting ideal gas with the constant adiabatic exponent γ. Pressure p_∞ and density ρ_∞ of the undisturbed gas are considered constants. As a result of numerical solution of this problem, tables of the gas-dynamic functions of a point explosion are calculated.

This article presents tables for the flat, cylindrical, and spherical explosions in an ideal gas for the values of γ equal to 1.3, 1.4, and 5/3 (for $\gamma = 1.3$ the data are given only in the cylindrical and spherical cases). The tables contain the basic functions describing the field of the gas flow over a wide time span. Also, some possible applications of these tables for solving other problems in the gas-dynamics of explosion and in hypersonic aerodynamics are explained in the introductory section.

The literature, already contains tables for an explosion in a gas with counterpressure [3]. However, these tables are only for the spherical case when $\gamma = 1.4$. Let us note also that we [4] first calculated the tables of the gas-dynamic functions for the initial stage of a point explosion, i.e., for sufficiently large pressure gradients at the shock wave front. The tables now being published are the natural continuation of the tables for the initial stage of an explosion.

The calculation of the current tables of gas-dynamic functions was accomplished on BESM-2 and BESM-3M computors of the Computation Center of the USSR Academy of Sciences. The numerical method of solving explosion problems, used to calculate the tabulated data, is comprehensively described in works [5, 6]. They give basic equations and a number of useful formulas and reductions, which are not reproduced here.

Taking the opportunity, the authors wish to express their gratitude to Ye. Bishimov for his assistance in compiling programs for the BESM-3M computor, to L. S. Bark, who supervised the checking of the tables for diversities, and to V. P. Karlikov for his valuable remarks under the editing of the manuscript.

1. METHOD OF CALCUATING DIMENSIONLESS GAS-DYNAMIC FUNCTIONS.

The functions of the explosion problem to be determined (pressure p, density ρ, velocity v, temperature T, total energy per unit volume E) depend upon geometric coordinate r and time t. Coordinate r in the plane case ($\nu = 1$) represents the distance from the plane of the explosion, in the cylindrical case ($\nu = 2$) – the distance from the axis of the explosion and in the spherical case ($\nu = 3$) – the distance from the point of the explosion. The law of motion for a shock wave, i.e., the dependence of the shock wave coordinate r_n upon time t, will also be determined when the problem is solved. The functions specified above depend parametrically upon the adiabatic exponent γ, the energy of the explosion E° and the initial values of the pressure p_{∞} and the density ρ_{∞}.

Various numerical methods are used for solving the problem of a point explosion. Thus, in the above-mentioned work [3], a special numerical method based on a scheme of finite differences was developed. The calculation of the gas-dynamic functions contained in the current tables was done by the integral relationship method.

The problem is easily solved in dimensionless variables. Let us introduce the following independent variables:

$$\xi = \left(\frac{r}{r_n}\right)^{\nu}; \quad q = \frac{a_{\infty}^2}{c^2}, \tag{1}$$

where c — the velocity of the shock wave; a_∞ — the speed of sound in a quiescent, which is expressed thus:

$$a_\infty = \sqrt{\gamma \frac{p_\infty}{\rho_\infty}}.$$

The range of change for the variables ξ and q is given by the inequalities $0 \leq \xi \leq 1$, $0 \leq q \leq 1$. In this range we will single out the central interval limited by lines $\xi = 0$ and $\xi = \xi_0(q)$, in which the solution is found with the help of the asymptotic formulas

$$\left.\begin{aligned} p &= p_0 + O(\xi^s); \\ \rho &= \rho_0\left(\frac{\xi}{\xi_0}\right)^{\frac{1}{\gamma-1}} + O\left(\xi^{\frac{2s\nu-1}{\nu}}\right); \\ v &= v_0\left(\frac{\xi}{\xi_0}\right)^{\frac{1}{\nu}} + O(\xi^{s+\frac{1}{\nu}}), \end{aligned}\right\} \qquad (2)$$

where

$$s = \frac{\nu + 2(\gamma - 1)}{\nu(\gamma - 1)}.$$

Here $p_0 = p(\xi_0, q)$, $\rho_0 = \rho(\xi_0, q)$, $v_0 = v(\xi_0, q)$ represent the values of the pressure, the density and the velocity at the boundary of the central interval, i.e., on line $\xi = \xi_0(q)$. To select a value for ξ_0, the Lagrangian coordinate of the particle is fixed so that in the initial stage of the explosion it is possible to determine the pressure with acceptable accuracy according to the appropriate asymtotic formula from (2).

The solution in the domain located between the line $\xi = \xi_0(q)$ and the shock wave $\xi = \xi_n = 1$ is formed by the method of integral relationships. In the n-th approximation this domain (Fig. 1) is

divided into n strips by n - 1 the intermediate lines:

$$\xi_i(q) = \frac{n-i}{n}\xi_0(q) + \frac{i}{n} \qquad (i = 1, 2, \ldots, n-1). \tag{3}$$

For convenience the new dimensionless unknown functions are introduced. The initial system of gas-dynamics equations is converted to dimensionless variables and is reduced to divergent form. Every equation of the obtained system is integrated with respect to ξ from the value $\xi = \xi_l$ $(l = 0, 1, \ldots, n - 1)$ to the shock wave $\xi = \xi_n = 1$, which leads to a system integral relationships. The approximation of the intergrands by Lagrange interpolation polynomials with interpolation points on lines $\xi = \xi_k(q)$ $(k = 0, 1, \ldots, n)$ gives an approximating system of ordinary differential equations with respect to q for determining the unknown gas-dynamic functions on lines $\xi = \xi_l(q)$ and the dimensionless coordinate R_n of the shock wave.

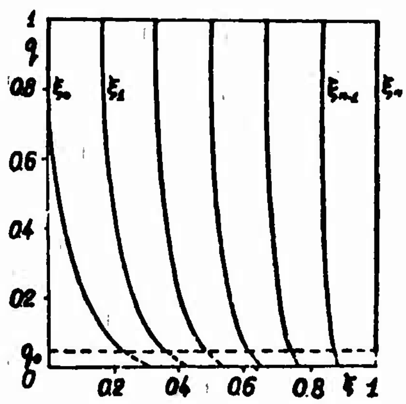

Fig. 1.

The initial equations and conditions of problem also make it possible to derive differential equations for determining the value of ξ_0 and of dimensionless time τ as functions of the variable q introduced above. Time t and the shock wave coordinate r_n are connected with the values τ and R_n in the following manner:

$$\tau = t^\circ \tau ; \quad r_n = r^\circ R_n , \tag{4}$$

where dynamic length r° and time t° are expressed thusly:

$$r^\circ = \left(\frac{E^\circ}{p_\infty}\right)^{\frac{1}{\nu}} ; \quad t^\circ = r^\circ \left(\frac{\rho_\infty}{p_\infty}\right)^{\frac{1}{2}} . \tag{5}$$

For a complete approximating system of differential equations the Cauchy problem is numerically solved with the initial data at a certain small value $q = q_0$ taken from the tables in [4], which contain the solutions of self-simulating and linearized explosion problems.

The tables presented here were calculated by dividing the intergration range into the eight strips, i.e., when n = 8, while in the case of constructing an approximating system of ordinary differential equations, the integrads in the integral realizations of the problem were replaced by two 4th order joining interpolation polynomials. Besides the basic calculation, when n = 8, calculations of lower approximations when n = 1, 2 and 4 were also made; in all these cases different forms of approximating polynomials were used. Comparison of the obtained results characterized the accuracy and practical convergence of the method of integral relationships in the explosion problem.

To check the accuracy of the numerical solution, the integral laws of conservation of mass and energy in the total volume of the moving gas were also used. Let us designate by ε_{1n} the relative mass error, which is equal to ratio of the difference between the calculated mass of the moving gas and the initial mass of gas in the volume limited by the shock wave front to the initial mass of the gas in this same volume. We will designate as ε_{2n} the ratio of the difference between the calculated total energy of the perturbed gas and the initial total energy in the previously mentioned volume of gas to the initial total energy of the gas in this same volume. The quantity ε_{2n} gives the relative energy error.

Furthermore, for initial value $q = q_0$, the error ε_{3n} was checked by the function $\psi = (qp/p_\infty)\ 1/\gamma$, conditioned by the selection of the central interval $\xi_0(q_0)$. The quantity ε_{3n} is introduced in the following manner:

$$\varepsilon_{3n} = \frac{\psi(\xi_0, q_0) - \psi(0, q_0)}{\psi(0, q_0)}.$$

During the assignment of the initial data the values q_0 and $\xi_0(q_0)$ were selected so that the conditions

$$|\varepsilon_{1n}(q_0)| < \varepsilon^{\circ}_{1n}; \quad |\varepsilon_{2n}(q_0)| < \varepsilon^{\circ}_{2n}; \\ |\varepsilon_{3n}(q_0)| < \varepsilon^{\circ}_{3n}.$$

would be satisfied at small q. Here, ε°_{1n}, ε°_{2n}, ε°_{3n} – assigned numbers, whose selection is determined both by the requirement for their smallness and by the conditions of the proximity of the numerical solution to the linearized solution in the vicinity of $q = q_0$ and the absence in the region of the small q of a physically unreal oscillation in the solution at large n ($n > 4$).

In the conducted calculations, the value of q_0 was always taken as equal to 0.05. The values of the initial errors ε_{1n}, ε_{2n}, ε_{3n} for all the calculated cases are shown in the following table.

	$\nu = 1$		$\nu = 2$			$\nu = 3$		
γ	1,4	5/3	1,3	1,4	5/3	1,3	1,4	5/3
ε_{1n}	0,043	0,007	0,014	0,009	0,003	0,016	0,008	0,009
ε_{2n}	0,021	−0,002	−0,001	−0,002	0,006	0,004	−0,002	0,003
ε_{3n}	0,078	0,028	0,023	0,025	0,013	0,032	0,026	0,078

Let us note that the errors in the initial data fade rather rapidly during calculation.

Internal checking of the accuracy of the calculation scheme was additionally accomplished by computing the dimensionless pressure using the equation of conservation of total energy and by using the equation of the conservation of entropy in a particle of gas. The relative divergence in the pressure values determined by two such methods was less than 0.01.

Curves of ε_{1n} as functions of q for $n = 8$ at various γ and with $\nu = 1$ (solid line), $\nu = 2$ (crosses), and $\nu = 3$ (circles) are presented in Fig. 2. As is evident, the integral error ε_{1n} in the basic component of the interval of change comprises from 0.002 to 0.02. As for the absolute value of ε_{2n}, it was maximum at the beginning of the calculation, i.e., when $q = q_0$, and then it dropped to values close to zero. One should still keep in mind that the major characteristics of an explosion (the law of motion for the shock wave and the pressure distribution near the center of the explosion) are derived with greater accuracy than the field of the gas dynamic functions. In the region of extreme values of q the accuracy of the calculated solution is reduced. However, for the initial stage of the explosion ($q \leq 0.10$) it is possible to use the linearized solution in [4], and with long times ($q > 0.90$) the asymptotic laws of the fading of shock waves can be used.

The evaluations of accuracy indicated above made it possible in the greater part of the tables to present the values of the functions to three decimal places; in certain cases the third place is not completely accurate.

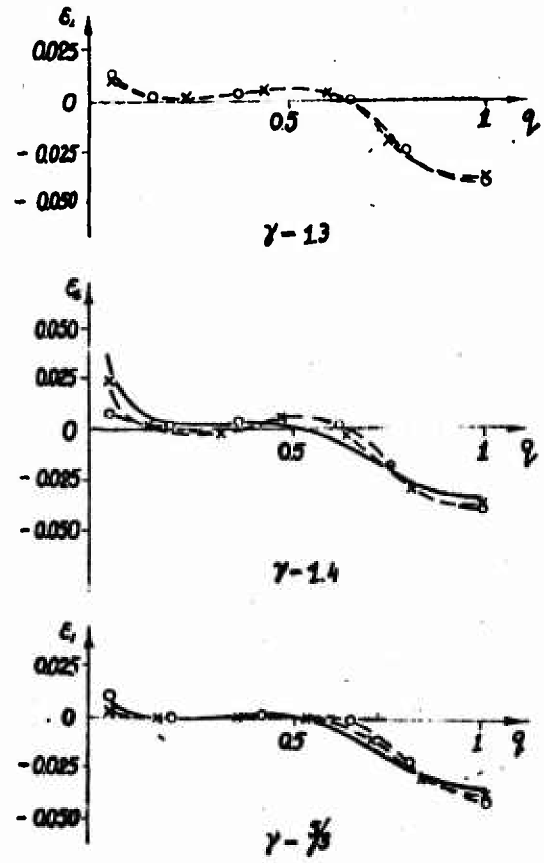

Fig. 2.

2. THE STRUCTURE OF TABLES AND FORMULAS FOR DETERMINING THE PHYSICAL PARAMETERS OF AN EXPLOSION

The tables contain the numerical solution of the problem of a point explosion in an ideal gas with counterpressure. They encompass the plane ($\nu = 1$), cylindrical ($\nu = 2$) and the spherical ($\nu = 3$) cases at adiabatic exponent values of $\gamma = 1.3$ (only for cases $\nu = 2$ and $\nu = 3$), $\gamma = 1.4$ (air) and $\gamma = 5/3$ (monatomic gas). The values of the basic gas-dynamic functions are given as functions of the variable q which characterizes time.

The argument of q varies from q = 0.05 to q = 0.90 with constant step of Δq = 0.05. With q = 0.05 and q = 0.10 data are given for the linearized solution. The values when q = 0 can be obtained from the tables in [4] for the self-similar solution.

At the beginning of the tables dimensionless time τ, the dimensionless shock wave coordinate R_n and the boundary of the central interval ξ_0 depending upon q are presented. The following relative functions, selected from considerations of the scale of the tabulated data, are given in the same place:

$$\frac{p_n}{p_\infty}q, \quad \frac{\rho_n}{\rho_\infty}, \quad \frac{v_n}{c}, \quad \frac{T_n}{T_\infty}q, \quad \frac{E_n}{p_\infty}q.$$

These functions give the values for pressure, density, velocity, temperature and energy directly behind the shock wave front. The latter values do not depend upon ν and are expressed by these formulas:

$$\left.\begin{aligned} &p_n = \frac{2\gamma-(\gamma-1)q}{(\gamma+1)q}p_\infty; \qquad \rho_n = \frac{\gamma+1}{\gamma-1+2q}\rho_\infty; \\ &v_n = \frac{2c}{\gamma+1}(1-q); \quad T_n = \frac{p_n}{R_\Gamma \rho_n}; \quad E_n = \frac{p_n}{\gamma-1} + \frac{\rho_n v_n^2}{2}. \end{aligned}\right\} \qquad (6)$$

In accordance with definition (1), the velocity c of the shock wave will be

$$c = \frac{a_\infty}{\sqrt{q}} = \sqrt{\frac{\gamma p_\infty}{q\rho_\infty}}. \qquad (7)$$

In the formulas in (6) R_Γ designates the gas constant.

The basic contents of tables are a description of the fields of the gas-dynamic functions, which for the sake of convenience are

related to the corresponding values of (6) and (7) at the shock wave front. For the pressure, density, velocity, temperature and energy the following relative variables are introduced:

$$\frac{p_l}{p_n}, \quad \frac{\rho_l}{\rho_n}, \quad \frac{v_l}{c}, \quad \frac{T_l}{T_n}, \quad \frac{E_l}{E_n}.$$

The tables give the values of these functions on lines $\xi = \xi_l(q)$ (l = 0, 1, ..., n - 1), i.e., at the boundary of the central interval $\xi = \xi_0(q)$ and on the intermediate lines $\xi = \xi_1(q)$ determined by equation (3). The values of the functions at points ξ that do not coincide with the end-points $\xi = \xi_l$ are found by interpolation. Here it is advantageous to use an interpolation polynomial not of the power n = 8, but of smaller powers (for instance, the second or third), while conducting local interpolation at the nearest nodal points, which is entirely sufficient in terms of the accuracy of the calculated solution.

Let us note certain peculiarities in the calculation of gas-dynamic functions inside the central interval when $0 \leq \xi < \xi_0$.

Since at the center of the explosion when $\xi = 0$, the velocity and the density are equal to zero, to compute the values v/c and ρ/ρ_n on the segment $[0, \xi_0]$ it is possible to either use quadratic interpolation over the known values of these functions at the points $\xi = 0$, $\xi = \xi_0$ and $\xi = \xi_1$ or use the asymptotic formulas in (2).

As for the pressure, its precise value at the center of the explosion is unknown, and here several methods of determining this value approximately are possible. Let us introduce the designation $h = p/p_n$, where this ratio of the pressures when $\xi = 0$ we will note as $h_0^* = h(0, q)$.

For rough estimation in conformity with (2), it is possible to assume that $h_0^* = h_0$. To find the pressure inside the central interval

it is also possible to use standard extrapolation over the values of h at the points $\xi = \xi_0$ and $\xi = \xi_1$. However, with a large value of the central interval this method gives the value of the pressure at $\xi = 0$ with a high degree of error. A more precise extrapolation for calculating h in the center of the explosion can be conducted in oonformity with asymptotics in (2) using the following formula:

$$h_0^{*} = h_0 - \frac{h_{01} - h_0}{\xi_{01}^{s} - \xi_0^{s}} \xi^{s},$$

where $h_{0\,1}$ is the value of h at the point $\xi = \xi_{0\,1}$ belonging to the segment $[\xi_0, \xi_1]$.

The determination of dimensional physical parameters with assigned specific values of the explosion energy E°, the pressure p_{∞} and the density ρ_{∞} of the undisturbed gas is done from the tabulated data without without special labor. Time t and the shock wave coordinate r_n are found from formulas (4) and (5), while the geometric coordinate r of any point is expressed through ξ in the following manner:

$$r = r_n \xi^{1/\nu}.$$

The basic physical parameters in dimensional form are calculated according to the simple relationships

$$p = \frac{p_{\infty}}{q}\left(\frac{p_n}{p_{\infty}} q\right)\left(\frac{p}{p_n}\right); \qquad \rho = \rho_{\infty}\left(\frac{\rho_n}{\rho_{\infty}}\right)\left(\frac{\rho}{\rho_n}\right);$$

$$v = \sqrt{\frac{\gamma p_{\infty}}{q \rho_{\infty}}}\left(\frac{v}{c}\right); \qquad T = \frac{p_{\infty}}{q R_r \rho_{\infty}}\left(\frac{T_n}{T_{\infty}} q\right)\left(\frac{T}{T_n}\right);$$

$$E = \frac{p_{\infty}}{q}\left(\frac{E_n}{p_{\infty}} q\right)\left(\frac{E}{E_n}\right),$$

where the functions presented in the tables are shown in brackets.

It is necessary to further note that the numerical solution of the explosion problem changes quite smoothly as a function of value of the adiabatic exponent γ in the range of the examined values of γ. This fact makes it possible with the help of interpolations to use the given tables for other cases, in which the adiabatic exponent γ is different from the tabulated values.

3. CALCULATION OF CERTAIN ADDITIONAL CHARACTERISTICS OF A GAS FLOW

The given tables can also be used for computing certain supplementary parameters of a gas flow during an explosion, which are not directly incorporated in these tables. Let us present a number of examples of such an application of the tables.

A. Determination of Flow Parameters at Fixed Points of Space

Calculating the change in flow parameters at fixed points in space is done in the following manner. Suppose that a point is examined that has a fixed dimensionless geometric coordinate $R = R_*$, where $R = r/r^0$. The perturbed motion of the gas at this point is initiated from that instant τ_* (or the corresponding values q_*), when the shock wave arrives at this point, i.e., when $R_n(q_*) = R_*$. For all subsequent q the following values are determined:

$$\xi_*(q) = \left[\frac{R_*}{R_2(q)}\right]^{\nu}.$$

For these ξ_*, using the tabulated values of the functions at the endpoints $\xi = \xi_l$, the unknown functions at the considered point with the coordinate R_* are found with the help of interpolation.

B. Determination of the Pulses of Excess Pressure and Velocity Pressure

Having assigned a certain value for coordinate R_*, using the procedure described above, let us find in this point of space the value of the dimensionless pressure $P = p/p_\infty$ for a sufficiently large number of instants τ. In terms of the found values of $P(R_*, \tau) = P_*$ it is possible to compute the complete dimensionless pulse of the excess pressures

$$J_p = \int_{\tau_*}^{\tau_K} (P_* - 1) d\tau, \tag{8}$$

and also the positive pulse

$$J_p^+ = \int_{\tau_*}^{\tau_K} (P_* - 1) d\tau \quad \text{when} \quad P_* - 1 > 0.$$

Here τ_* is the time of the shock wave arrival at a given point; τ_K is the final calculated time. The dimensional pulse I_p is connected with the dimensionless pulse J_p by the relationship

$$I_p = \int_{t_*}^{t_K} (p_* - p_\infty) dt = J_p t^\circ p_\infty .$$

By a similar method it is possible to calculate the dimensionless pulses of the velocity pressure J_v by the formula

$$J_v = \int_{\tau_*}^{\tau_K} \frac{\rho_*}{\rho_\infty} \left(\frac{v_*}{c}\right)^2 \frac{d\tau}{2q}, \tag{9}$$

where the connection with the corresponding dimensional pulse will be

$$I_v = \int_{t_*}^{t_K} \frac{\rho_* v_*^2}{2} dt = J_v t^\circ \rho_\infty a_\infty^2 .$$

It is natural that the integrals in expressions (8) and (9) should be calculated by suitable quadrature formulas; for example, during the variable step of integration by the trapezoidal rule. In this calculation it is possible to also determine the action time τ^+ for the positive pressure phase, using the equality

$$\tau^+ = \bar{\tau} - \tau_*,$$

where $\bar{\tau}$ is the time for transition of the excess pressure to the negative phase.

During the numerical solution of the explosion problem, the pulses in the cylindrical and spherical cases for $\gamma = 1.4$ were calculated as an example. The variation in the pulses J_p^+ and J_υ and also in time τ^+ and $\bar{\tau}$ that depend on coordinate R for $\nu = 2$ is presented in the form of curves in work [6]. Also given there, for the cylindrical case, are curves of dimensionless pressure P, which depends on τ, for a series of fixed points of space.

C. Determining the Connection Between Lagrange and the Euler Coordinates

Let us designate by r_0 the beginning coordinate for a particle of gas. Let us introduce a dimensionless Lagrangian coordinate in the form of $\eta = (r_0/r^\circ)^\nu$, where r° is the characteristic length, determined by expression (5). Then the connection between Lagrangian coordinate η and the Eulerian coordinate ξ is realized with the help of a continuity equation in this form:

$$\eta = R_n^\nu \left(1 - \int_\xi^1 \frac{\rho}{\rho_\infty} d\xi\right).$$

By using the tabulated data, it is possible to find the connection between η and ξ for a fixed τ.

4. USE OF THE TABLES IN SOLVING CERTAIN GAS-DYNAMIC PROBLEMS

A. Determining the Parameters of a Point Explosion in a Steady Forward Gas Flow

Let us assume that an explosion occurs in a gas which is moving with constant velocity U. If we wish to examine the process of the development of the explosion in a fixed coordinate system, it is necessary to recalculate the solution by using the Galileo-Newton transform. Let us select a certain fixed system of Cartesian rectangular coordinates $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ with the origin in the center of the explosion at $t = 0$. Let us designate by U_1, U_2, U_3 projections of the velocity vector $\bar{U}$ to the coordinate axes, and by ϑ_1, ϑ_2, ϑ_3 the corresponding projections of the velocity vector of the gas.

Then during examination of the gas flow in the fixed coordinate system its parameters can be computed by using the conversion formulas

$$\hat{x}_j = x_j + U_j t; \quad \hat{v}_j = v_j + U_j; \quad \hat{p} = p; \quad \hat{\rho} = \rho,$$

here v_j, p, ρ are the values of the components of velocity, pressure and density in the coordinate system x_j associated with the forward moving gas. The solution of the problem in this moving coordinate system corresponds to the data contained in the tables. Thus, to determine the parameters of an explosion in a forward gas flow, one should only conduct a simple recalculation of these tabulated data. When conducting such a recalculation in the case of a cylindrical or a spherical explosion, it is necessary to take into account the geometric relationships resulting from the connection between Cartesian and cylindrical or Cartesian and spherical coordinates.

B. Initial Stage of the Reflection of a Flat Blast from a Parallel Plane Wall and of Cylindrical and Spherical Waves from a Concentric Cylindrical or Spherical Wall, Respectively

Let us examine the question of determining the pressure, density and velocity of a gas behind the front of a reflected shock wave for moments of time close to the moment of approach of the wave to a wall (designated by t_*). On the strength of the assumed geometry of the wall, the reflection of the shock wave from it will be a normal reflection, but the flow behind the front of the reflected wave will be one-dimensional with plane, cylindrical, or spherical symmetry. In this case, the pressure p_*, the density ρ_* and the temperature T_* in the reflected wave are found by using the well-known formulas in gas dynamics (see, for example, [7]):

$$\frac{p_*}{p_\infty} = 2\left(\frac{p_n}{p_\infty} - 1\right) + \frac{\left(\frac{p_n}{p_\infty} - 1\right)^2}{\frac{\gamma - 1}{\gamma + 1}\frac{p_n}{p_\infty} + 1}; \tag{10}$$

$$\frac{\rho_*}{\rho_\infty} = \frac{\gamma \frac{p_n}{p_\infty}\frac{\rho_n}{\rho_\infty}}{(\gamma - 1)\frac{p_n}{p_\infty} + 1}; \qquad \frac{T_*}{T_\infty} = \frac{p_*}{p_\infty}\frac{\rho_\infty}{\rho_*}. \tag{11}$$

For the value of the initial velocity of the reflected wave c_*, we have the expression

$$\frac{c_*}{a_\infty} = \sqrt{\frac{2}{\gamma}}\left[(\gamma - 1)\frac{p_n}{p_\infty} + 1\right]\left[(\gamma + 1)\frac{p_n}{p_\infty} + (\gamma - 1)\right]^{-1/2}. \tag{12}$$

Since the wall is stationary, on the strength of the boundary condition on it the velocity behind the front of the reflected shock wave v_* will be small for moments of time close to t_*.

If we assign the distance r_* from the point of impact to the wall and the value of characteristic length r°, then, knowing the dimensionless coordinate R_* let us find, according to the value $R_n = R_*$ with the help of the tables, the dimensionless time of the approach of the wave to the wall τ_* and by it time t_*, and also the corresponding values of q, p_n/p_∞ and ρ_n/ρ_∞. Furthermore, by using formulas (10)-(12), the basic parameters of the reflected shock wave for moments of time close to t_*.

Let us note that formulas (10)-(12) for the problem in question, strictly speaking, are accurate only at the moment the shock wave directly reflects from the wall, since they were derived for a uniform flow behind the incident wave. However, these relationships will be approximately fulfilled for those moments of time when the variability of the parameters of the flow behind the incident wave can be disregarded.

C. Initial Stage of Regular Reflection for a Plane, Cylindrical or Spherical Blast from a Flat Surface

Let us first examine the plane case. Suppose that a plane blast strikes an absolutely rigid flat wall so that the angle between the plane of the shock wave and the wall is different from zero and equal to α. Then, for the moments of time close to the moment the wave collides with the wall, from relationships on the shock wave and the boundary conditions on the wall for the velocity of the gas, it is possible to derive the analytical dependences between the parameters of the incident and reflected waves (see [7]).

Let us designate by α_*, p_*, ρ_* respectively the reflection angle, the pressure and the density behind the reflected wave at the moment of time directly after the reflection, and let us introduce the following parameters:

$$k_\infty = \frac{p_\infty}{p_n}; \quad k_* = \frac{p_*}{p_n}; \quad \omega = \operatorname{tg}\alpha; \quad \omega_* = \operatorname{tg}\alpha_* .$$

If we assign angle α and the pressure ratio h_∞ in the moment the wave meets the reflecting plane, then to determine ω_* we have the quadratic equation

$$m[(1-\mu)^2-(\omega-\omega_*)^2-(\mu+\omega\omega_*)^2]+m^2(1-\mu)^2(\omega-\omega_*)-\omega+\omega_*=0, \tag{13}$$

where

$$\mu=\frac{\gamma-1}{\gamma+1}; \quad m=\frac{(1-h_\infty)\,\omega}{1+\mu h_\infty+(\mu+h_\infty)\,\omega^2}.$$

The value of the relative pressure h_* behind the reflected wave is expressed thusly:

$$h_*=\frac{m(1+\mu\omega_*^2)+\omega_*}{\omega_*-m(\mu+\omega_*^2)}. \tag{14}$$

The ratio of the densities ρ_*/ρ_n and of the temperatures T_*/T_n can be found with the help of formulas (11). Since the point of intersection of the incident and reflected waves moves along the plane, the velocity of the reflected shock wave c_* at the point of reflection can be obtained from the relationship

$$c_*=c\,\frac{\sin\alpha_*}{\sin\alpha}. \tag{15}$$

By using the found values c_*, ρ_*/ρ_n and the law of conservation of mass during the transition through the explosion surface, it is possible to determine the normal velocity component of the gas behind the reflected wave, v_*. Viscous component is found by the condition of its continuum during the transition through the discontinuity. Let us note that the lack of real roots of equation (13) gives angular values to which the irregular (Mach) reflection sets in.

Let us now examine the case of the reflection of a cylindrical or a spherical wave. Suppose that an explosion takes place at a distance L from the reflecting plane Π (in the cylindrical case for simplicity let us consider that the line of explosion is parallel to plane Π). Let us designate as α the angle at which a cylindrical or a spherical wave will approach the plane Π at a certain instant t_* after an explosion. From the geometry (Fig. 3), it is evident that angle α is equal to the angular between the perpendicular to plane Π and the radius-vector drawn from the center of the explosion O to the reflection point of the wave O' at the instant in question, t_*.

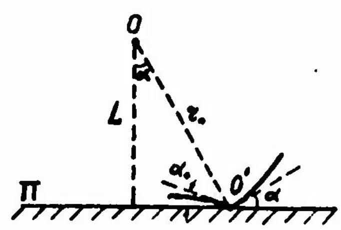

Fig. 3.

By applying the results of solving the problem of the reflection of a plane wave to the reflection of an element of a cylindrical or a spherical wave, it is possible to determine the gas parameters directly at the moment the wave element reflects from the plane. Having designated as H the dimensionless quantity L/r°, we will have the equality

$$\cos\alpha = \frac{L}{r_*} = \frac{H}{R_*}. \tag{16}$$

Let us draw our interest to the parameters of the reflected wave at point O' of plane Π at distance r_* from the center of the explosion. If the parameter r° is known, then by taking $R_* = r_*/r^\circ$, it is possible to calculate angle α by formula (16). With the help of the tables in terms of the value $R_n = R_*$, let us find values

q_*, τ_* and the ratios p_n/p_∞, ρ_n/ρ_∞. If with the derived p_n/p_∞ angle α corresponds to the condition of a regular reflection, then, by using the designation introduced above and using formulas (11) and (13)-(15), let us calculate the parameters of the reflected wave at instants close to t_*.

Furthermore, suppose that the dependence between p_n/p_∞ and the critical angle of a regular reflection is known. On the basis of theoretical analysis, such data follow from formulas (13)-(16); they are given, for example, in book [7]. Then, for a given value L, it is possible to find the boundary of the zone a regular reflection on plane П.

D. Determining the Parameters of a Flow During an Explosion on a Flat Boundary Between a Gas and a Solid or a Liquid Medium

Let us assume that the upper half-space is occupied by a gas, and the lower – by some other medium. Let us examine first the idealized case when the lower half-space is occupied by an absolutely rigid body. To determine, with the help of the tables in this work, the parameters of a flow during the explosion of a charge at the boundary of the gas with an absolutely solid plane, one should take the value $\frac{1}{2}E°$ in all cases instead of the value E°.

Suppose that the lower half-space is now occupied by a deformable medium, but the flow has such a character that the processes occurring in the lower half-space very weakly affect the gas flow in the upper half-space. Furthermore, let us assume that the time of the shock wave arrival at some fixed point of the upper half-space is known (for instance, this time is measured in the experiment). As such a point it is convenient to take any point arranged on a line, perpendicular to the boundary plane and passing through the center of the explosion in the cylindrical and spherical cases.

Then it is possible to approximately calculate those parts of the explosion energy E° which passed into the upper and lower half-spaces and to find the gas flow parameters in the upper half-space. Actually, in terms of the coordinate of fixed point r_* and the time of the shock wave arrival t_*, by keeping in mind formulas (4), (5) and by using from the tables the dependence of R_n upon τ_*, it is possible to determine the energy E_1° which was released into the upper half-space It is evident that the remaining part of energy E_2° will go into the lower half-space, since

$$E_1^{\circ} + E_2^{\circ} = E^{\circ}. \tag{17}$$

Knowing energy E_1°, from the tables it is possible to obtain all the gas-dynamic parameters of interest and to approximately describe the explosion process in the upper half-space.

Let us note that a similar, but purely experimental approach to finding the energy distribution between the two half-spaces was examined in work [8].

E. Plane Explosion at the Interface of Two Identical Gases Having Equal Initial Pressures, but Different Initial Densities

Let us discuss the question of using the tables to solve the problem of determining gas-dynamic parameters during a plane explosion at the interface of the gases with the initial parameters γ, p_∞, $\rho_{\infty 1}$ and γ, p_∞, $\rho_{\infty 2}$.

Let us assume that the Lagrangian coordinate of the contact surface is $\eta_* = 0$. From the condition of the equality of the pressures on this surface, we have

$$P_1(0, \tau) = P_2(0, \tau), \tag{18}$$

where P_1 and P_2 are dimensionless pressures in the first and second half-spaces, respectively. Let us take now as functions of P_1 and P_2 the dependences obtained for a homogeneous medium. Then equality (18) will be satisfied, if $\tau_1 = \tau_2$, for any t. Hence, taking into account formulas (5), let us obtain the relation between the fractions of energy E_1° that went out into first half-space, and fractions of E_2° that were released in the second half-space, and namely:

$$E_1^\circ = E_2^\circ \sqrt{\frac{\rho_{\infty 2}}{\rho_{\infty 1}}}, \tag{19}$$

in this case, formula (17) naturally takes place.

Thus, if at the interface of identical gases there occurred a plane explosion with energy E°, then determining the physical characteristics of the flow in both half-spaces can be done from the given tables while taking into account relationships (17)-(19).

There is a possibility of broadening the circle of problems which are resolved with the help of the results of calculating a point explosion in a homogeneous gas. Thus, by using the principle of flat cross sections, in a number of cases it is possible to approximately find the parameters of an explosion in a heterogeneous medium. If, for example, in the case of a cylindrical explosion the initial density is slightly changed in the direction of the axis of the explosion, then here, on the strength of the principle of flat cross sections, it is possible to conduct calculations for the parameters of the explosion by a local one-dimensional theory. The principle of flat cross sections also makes it possible to study the explosion along a line, which is different from a straight line, but whose curvature is everywhere small. Another widely known and important example of using of the principle of flat cross sections will be examined in the following section.

F. Supplement to Problems of Hypersonic Flow Around Thin Blunted Plates and Cylinders.

Between the nonstationary problem of an explosion and the problem of steady flow over bodies at hypersonic velocities a known analogy exists, which is based upon the principle of flat cross sections [9]. Thus, with the help of the calculated solution for plane and cylindrical explosions, it is possible to approximately determine the hypersonic flow around thin plates and cylinders with small blunting. In this case, the parameters of the nonstationary problem are replaced by the stationary parameters, with t and E° respectively replaced by xU_{∞} and $\frac{1}{2}c_x\rho_{\infty}U_{\infty}^2(\pi/4)^{\nu-1}d^{\nu}$. Here the following designations are assumed: x — the coordinate along the axis of the body, which is measured from its tip; y — the coordinate in a direction perpendicular to the surface of the plate or to the axis of the streamlined cylinder; d — the characteristic linear, cross section size of blunt part; U_{∞} — the velocity of the incoming gas flow, which is directed along the axis x; c_x — the resistance coefficient of blunt part, which is calculated per unit area of the cross-section of the body and referred to as the velocity pressure.

During such a change, the connection between the dimensionless values x/d, y/d and τ, R is given by the relationships

$$\left.\begin{aligned} \frac{x}{d} &= \left(\frac{\pi}{4}\right)^{\frac{\nu-1}{\nu}}\left(\frac{c_x\gamma^{\frac{\nu+2}{2}}}{2}\right)^{\frac{1}{\nu}} M_{\infty}^{\frac{2-\nu}{\nu}}\tau; \\ \frac{y}{d} &= \left(\frac{\pi}{4}\right)^{\frac{\nu-1}{\nu}}\left(\frac{c_x\gamma}{2}\right)^{\frac{1}{\nu}}\cdot M_{\infty}^{\frac{2}{\nu}}R, \end{aligned}\right\} \qquad (20)$$

where $M_{\infty} = U_{\infty}/a_{\infty}$ — the Mach number of the incident flow. Using formulas in (20), it is possible with the help of the given tables to determine the gas flow parameters near a plate and a cylinder at sufficient distance from the blunt portion. Specifically, it is possible to obtain the pressure distribution on body p/p_{∞}, the shape of the leading shock wave (i.e., the dependence of y/d on x/d) and the values of the basic gas-dynamic functions behind it.

The relationships given above in sections B and C (see also [7]) make it possible to calculate the parameters for the gas directly behind the front of the reflected shock wave which appears upon the regular reflection of the leading shock wave (being formed in front of the body) from a flat rigid surface. Such a reflection can take place during a hypersonic flow over a body in channels or during the motion of thin blunt bodies in a gas near solid boundaries.

BIBLIOGRAPHY

1. Седов Л.И. Методы подобия и размерности в механике. 6-е изд. М., Изд-во "Наука", 1967.

2. Коробейников В.П., Мельникова Н.С., Рязанов Е.В. Теория точечного взрыва. М., Физматгиз, 1961.

3. Охоцимский Д.Е., Кондрашова И.Л., Власова З.П., Казакова Р.К. Расчет точечного взрыва с учетом противодавления. М., 1957 (Труды МИАН СССР. Т. 50).

4. Коробейников В.П., Чушкин П.И., Шароватова К.В. Таблицы газодинамических функций начальной стадии точечного взрыва. "Сообщения по вычислительной математике", вып. 2. М., ВЦ АН СССР, 1963.

5. Коробейников В.П., Чушкин П.И. Метод расчета точечного взрыва в газах. "Докл. АН СССР", 1964, 154, № 3, 549-552.

6. Коробейников В.П., Чушкин П.И. Плоский, цилиндрический и сферический взрыв в газе с противодавлением. В кн.: "Неустановившиеся движения сжимаемых сред с взрывными волнами". М., 1967, 4-34 (Труды МИАН СССР. Т. 87).

7. Курант Р., Фридрихс К. Сверхзвуковое течение и ударные волны. М., Изд-во иностр. лит., 1950.

8. Алексеенко В.Д. Экспериментальное исследование распределения энергии при контактном взрыве. "Физика горения и взрыва", 1967, 3, № 1, 152-155.

9. Черный Г.Г. Течения газа с большой сверхзвуковой скоростью. М., Физматгиз, 1959.

TABLES OF GAS-DYNAMIC FUNCTIONS FOR POINT EXPLOSION

$\nu = 1$ $\gamma = 1.4$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty}q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n}{c}$	$\frac{T_n}{T_\infty}q$	$\frac{E_n}{p_\infty}q$
0.05	0.0021	0.0163	0.600	1.158	4.800	0.792	0.241	5.002
10	0066	0358	476	1.150	4.000	750	288	4.450
15	0151	0641	379	1.142	3.429	708	333	4.058
20	0277	0999	304	1.133	3.000	667	378	3.767
0.25	0.0446	0.142	0.249	1.125	2.667	0.625	0.422	3.542
30	0687	196	202	1.117	2.400	583	465	3.363
35	102	266	163	1.108	2.182	542	508	3.219
40	150	358	129	1.100	2.000	500	550	3.100
45	219	483	0995	1.092	1.846	458	592	3.001
0.50	0.318	0.653	0.0754	1.083	1.714	0.417	0.632	2.917
55	465	0.892	0559	1.075	1.600	375	672	2.845
60	0.687	1.24	0404	1.067	1.500	333	711	2.783
65	1.04	1.76	0285	1.058	1.412	292	749	2.730
70	1.61	2.59	0194	1.050	1.333	250	788	2.683
0.75	2.67	4.06	0.0123	1.042	1.263	0.208	0.825	2.643
80	5.17	7.40	0068	1.033	1.200	167	861	2.607
85	11.8	16.0	0031	1.025	1.143	125	897	2.575
90	25.4	33.2	0015	1.017	1.091	083	932	2.547

$\nu = 1$ $\gamma = 1.4$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.423	0.442	0.470	0.506	0.557	0.625	0.718	0.841
10	388	402	423	456	506	578	683	827
15	364	384	402	432	488	555	662	809
20	354	380	396	426	473	548	654	802
0.25	0.360	0.382	0.399	0.430	0.479	0.552	0.658	0.805
30	370	390	406	436	485	558	663	808
35	385	402	417	447	494	566	670	812
40	407	421	434	460	505	575	677	818
45	433	446	457	479	519	585	684	822
0.50	0.470	0.479	0.487	0.504	0.536	0.597	0.692	0.826
55	513	519	524	534	558	611	701	831
60	561	565	567	572	587	628	710	836
65	612	614	615	617	624	650	720	841
70	664	666	666	667	669	680	732	846
0.75	0.717	0.720	0.720	0.720	0.720	0.720	0.749	0.848
80	771	774	774	774	774	773	778	844
85	826	830	829	830	829	830	828	851
90	881	886	886	886	885	886	885	890

$\nu = 1$ $\gamma = 1.4$

q	$\frac{\rho_0}{\rho_\pi}$	$\frac{\rho_1}{\rho_\pi}$	$\frac{\rho_2}{\rho_\pi}$	$\frac{\rho_3}{\rho_\pi}$	$\frac{\rho_4}{\rho_\pi}$	$\frac{\rho_5}{\rho_\pi}$	$\frac{\rho_6}{\rho_\pi}$	$\frac{\rho_7}{\rho_\pi}$
0.05	0.188	0.203	0.254	0.318	0.399	0.502	0.631	0.790
10	133	167	227	302	399	523	674	815
15	108	158	230	311	410	532	674	815
20	102	168	252	357	438	543	666	816
0.25	0.0984	0.165	0.281	0.377	0.473	0.570	0.686	0.828
30	0972	208	322	420	505	596	706	840
35	0981	242	370	460	535	621	725	849
40	101	292	421	498	566	644	739	858
45	104	353	475	537	594	662	751	864
0.50	0.110	0.429	0.530	0.575	0.620	0.678	0.761	0.869
55	116	497	584	614	646	695	771	874
60	124	571	638	653	675	712	780	879
65	131	641	688	696	709	732	789	882
70	139	704	737	740	748	758	800	888
0.75	0.147	0.761	0.784	0.784	0.789	0.790	0.813	0.889
80	154	812	830	828	832	832	835	886
85	162	859	873	870	874	876	875	891
90	170	902	916	912	916	918	917	919

$\nu = 1$ $\gamma = 1.4$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.406	0.445	0.485	0.529	0.575	0.624	0.677	0.733
10	292	335	381	430	483	542	606	676
15	205	249	296	346	404	469	542	622
20	142	184	229	279	338	407	485	572
0.25	0.099	0.138	0.181	0.230	0.287	0.355	0.436	0.527
30	066	100	138	183	238	307	388	482
35	040	067	100	141	193	261	344	438
40	022	043	068	102	151	217	299	395
45	011	024	043	070	112	174	256	352
0.50	0.004	0.012	0.024	0.043	0.077	0.134	0.214	0.310
55	001	005	011	023	048	096	172	268
60	000	002	004	010	024	062	132	227
65	000	000	001	003	010	033	093	186
70	000	000	000	001	002	012	057	146
0.75	0.000	0.000	0.000	0.000	0.000	0.001	0.025	0.103
80	000	000	000	000	000	-0.001	0.005	056
85	000	000	000	000	000	0.001	-0.001	017
90	000	000	000	000	000	001	0.000	008

$\nu = 1$ $\gamma = 1.4$

q	T_0/T_n	T_1/T_n	T_2/T_n	T_3/T_n	T_4/T_n	T_5/T_n	T_6/T_n	T_7/T_n
0.05	2.14	2.18	1.849	1.593	1.396	1.247	1.137	1.065
10	2.91	2.41	1.869	1.509	1.266	1.105	1.014	1.015
15	3.39	2.42	1.750	1.390	1.172	1.042	0.983	0.993
20	3.48	2.27	1.577	1.261	1.092	1.008	0.981	0.983
0.25	3.66	2.07	1.420	1.139	1.012	0.968	0.959	0.972
30	3.80	1.875	1.262	1.039	0.961	936	939	962
35	3.93	1.658	1.129	0.971	922	911	925	957
40	4.04	1.442	1.031	924	893	894	917	954
45	4.16	1.263	0.963	892	874	884	912	952
0.50	4.28	1.134	0.919	0.875	0.865	0.879	0.910	0.951
55	4.41	1.045	896	871	864	879	909	951
60	4.54	0.989	889	876	870	882	910	952
65	4.66	958	893	887	880	888	912	953
70	4.78	947	904	902	895	898	916	954
0.75	4.89	0.946	0.918	0.910	0.912	0.912	0.921	0.954
80	5.00	953	933	936	930	929	931	953
85	5.10	966	949	953	948	948	947	955
90	5.19	982	967	971	966	966	966	967

$\nu = 1$ $\gamma = 1.4$

q	E_0/E_n	E_1/E_n	E_2/E_n	E_3/E_n	E_4/E_n	E_5/E_n	E_6/E_n	E_7/E_n
0.05	0.245	0.284	0.312	0.354	0.411	0.493	0.609	0.766
10	251	272	296	332	388	473	597	756
15	259	276	295	326	377	460	583	756
20	266	291	303	333	384	462	579	752
0.25	0.286	0.304	0.319	0.352	0.401	0.476	0.591	0.760
30	307	322	339	369	417	492	603	768
35	332	344	360	389	435	507	618	777
40	361	372	386	411	454	524	631	786
45	395	405	416	437	475	541	644	793
0.50	0.437	0.445	0.452	0.468	0.500	0.559	0.657	0.802
55	484	491	495	505	529	579	671	810
60	537	541	543	548	563	602	685	818
65	593	595	596	598	605	630	700	825
70	650	652	652	653	654	665	713	834
0.75	0.706	0.709	0.709	0.709	0.709	0.710	0.738	0.830
80	764	767	767	767	767	766	771	837
85	822	826	826	825	825	826	823	847
90	880	884	883	883	883	884	883	838

$\nu = 1$ $\gamma = 5/3$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty} q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{\nu_n}{c}$	$\frac{T_n}{T_\infty} q$	$\frac{E_n}{p_\infty} q$
0.05	0.0029	0.0244	0.400	1.238	3.478	0.712	0.356	3.328
10	0090	0534	281	1.225	3.077	675	398	3.006
15	0196	0920	210	1.213	2.759	638	440	2.753
20	0356	141	167	1.200	2.500	600	480	2.550
0.25	0.0580	0.202	0.132	1.188	2.286	0.562	0.520	2.384
30	0889	278	106	1.175	2.105	525	559	2.246
35	132	377	0839	1.163	1.951	488	596	2.130
40	195	508	0657	1.150	1.818	450	633	2.032
45	283	682	0507	1.137	1.702	412	668	1.948
0.50	0.408	0.916	0.0386	1.125	1.600	0.375	0.703	1.875
55	599	1.26	0287	1.112	1.509	338	737	1.812
60	0.885	1.74	0208	1.100	1.429	300	770	1.757
65	1.34	2.48	0146	1.087	1.356	262	802	1.709
70	2.08	3.64	0100	1.075	1.290	225	833	1.667
0.75	3.49	5.78	0.0063	1.062	1.231	0.188	0.863	1.630
80	6.64	10.4	0035	1.050	1.176	150	893	1.597
85	15.0	22.2	0016	1.037	1.127	112	920	1.568
90	33.2	47.3	0008	1.025	1.081	075	948	1.543

$\nu = 1$ $\gamma = 5/3$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.365	0.379	0.401	0.434	0.483	0.554	0.657	0.801
10	349	357	374	402	448	520	628	787
15	344	352	366	393	438	508	620	782
20	346	356	369	396	440	511	622	783
0.25	0.354	0.363	0.375	0.400	0.445	0.518	0.627	0.784
30	367	374	387	412	456	528	638	792
35	384	390	402	426	470	540	647	799
40	405	410	421	444	484	552	656	804
45	433	437	446	465	502	566	666	810
0.50	0.466	0.469	0.476	0.491	0.522	0.580	0.677	0.816
55	506	509	513	523	547	596	687	822
60	551	553	555	561	576	616	698	828
65	604	600	602	605	614	639	709	834
70	658	652	653	653	658	670	723	839
0.75	0.714	0.705	0.706	0.706	0.708	0.711	0.740	0.840
80	770	761	762	761	763	762	770	839
85	828	818	819	818	820	821	819	846
90	888	877	878	877	878	879	879	885

$\nu = 1$ $\gamma = 5/3$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.148	0.186	0.239	0.303	0.383	0.484	0.613	0.780
10	104	155	218	293	384	497	638	798
15	0879	154	228	311	406	518	655	812
20	0854	169	258	347	442	546	672	824
0.25	0.0823	0.187	0.292	0.388	0.481	0.582	0.697	0.835
30	0813	215	334	434	522	613	720	848
35	0816	251	385	481	558	640	739	860
40	0830	300	441	525	591	665	756	868
45	0854	362	500	568	623	687	770	875
0.50	0.0884	0.432	0.556	0.607	0.650	0.705	0.782	0.881
55	0925	510	613	647	679	724	793	887
60	0969	586	666	685	708	741	802	891
65	102	656	714	724	740	760	811	895
70	107	720	760	764	774	784	822	900
0.75	0.112	0.775	0.803	0.803	0.811	0.813	0.834	0.900
80	118	825	846	843	849	850	855	900
85	123	868	885	881	887	888	887	904
90	128	908	924	919	925	925	925	929

$\nu = 1$ $\gamma = 5/3$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.224	0.270	0.318	0.370	0.427	0.490	0.559	0.633
10	142	190	240	294	354	422	499	584
15	093	137	185	239	299	368	449	540
20	064	104	148	198	255	324	406	500
0.25	0.043	0.080	0.118	0.162	0.217	0.285	0.366	0.460
30	028	059	093	133	183	248	329	422
35	018	042	070	105	151	213	292	386
40	010	028	050	079	120	179	256	348
45	005	018	033	056	092	146	220	311
0.50	0.002	0.010	0.021	0.037	0.066	0.115	0.136	0.276
55	001	005	011	022	043	084	151	239
60	000	003	006	011	024	056	117	203
65	000	000	002	005	012	032	084	166
70	000	000	001	002	005	014	053	131
0.75	0.000	0.000	0.000	0.000	0.002	0.004	0.025	0.092
80	000	000	000	000	000	000	006	052
85	000	000	000	000	000	001	000	018
90	000	000	000	000	000	001	000	004

$\nu = 1$ $\gamma = 5/3$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	2.46	2.04	1.677	1.432	1.262	1.145	1.068	1.027
10	3.36	2.30	1.717	1.375	1.168	1.045	0.985	0.986
15	3.91	2.29	1.602	1.262	1.079	0.982	946	964
20	4.06	2.11	1.433	1.139	0.997	936	925	951
0.25	4.31	1.936	1.285	1.032	0.925	0.890	0.900	0.940
30	4.52	1.744	1.156	0.950	874	862	886	934
35	4.70	1.554	1.044	887	842	843	876	930
40	4.88	1.368	0.954	844	819	830	868	927
45	5.07	1.207	892	820	806	824	866	926
0.50	5.26	1.085	0.855	0.809	0.802	0.822	0.866	0.927
55	5.47	0.997	837	809	805	825	867	928
60	5.69	943	834	820	814	831	870	929
65	5.91	914	843	836	830	840	874	931
70	6.13	906	859	856	850	855	880	933
0.75	6.34	0.910	0.879	0.879	0.873	0.874	0.887	0.933
80	6.54	923	901	903	898	898	901	932
85	6.74	942	925	928	924	924	923	935
90	6.93	966	950	954	950	950	950	952

$\nu = 1$ $\gamma = 5/3$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.204	0.223	0.245	0.279	0.331	0.410	0.533	0.716
10	213	224	240	268	316	394	518	703
15	228	235	248	275	320	395	520	715
20	244	254	263	289	335	408	530	722
0.25	0.265	0.272	0.282	0.307	0.350	0.425	0.543	0.727
30	288	292	304	330	372	444	561	739
35	314	317	330	353	395	464	578	752
40	344	347	358	379	417	484	594	761
45	379	382	391	409	443	506	611	772
0.50	0.419	0.422	0.428	0.442	0.471	0.528	0.628	0.782
55	466	469	472	482	504	553	646	792
60	518	519	522	527	542	579	662	802
65	577	573	574	578	586	610	680	812
70	637	630	631	632	636	648	701	822
0.75	0.698	0.690	0.690	0.691	0.693	0.695	0.724	0.826
80	760	750	751	751	752	752	760	828
85	822	812	813	812	813	814	812	839
90	885	874	876	874	876	877	876	882

$\nu = 2$ $\gamma = 1.3$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty} q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n}{c}$	$\frac{T_n}{T_\infty} q$	$\frac{E_n}{p_\infty} q$
0.05	0.0090	0.0898	0.435	1.124	5.750	0.826	0.196	6.297
10	0192	132	345	1.117	4.600	783	242	5.556
15	0318	174	283	1.110	3.833	739	290	5.064
20	0468	215	238	1.104	3.286	696	336	4.715
0.25	0.0639	0.256	0.200	1.098	2.875	0.652	0.382	4.454
30	0846	301	167	1.091	2.556	609	427	4.253
35	108	349	138	1.085	2.300	565	472	4.094
40	137	402	114	1.078	2.091	522	516	3.964
45	174	466	0904	1.072	1.917	478	559	3.857
0.50	0.220	0.543	0.0701	1.065	1.769	0.435	0.602	3.768
55	278	633	0529	1.059	1.643	391	644	3.692
60	352	745	0385	1.052	1.533	348	686	3.628
65	452	0.890	0265	1.046	1.438	304	727	3.572
70	596	1.09	0172	1.039	1.353	261	768	3.524
0.75	0.807	1.37	0.0105	1.033	1.278	0.217	0.808	3.481
80	1.13	1.79	0060	1.026	1.211	174	848	3.444
85	1.74	2.55	0029	1.020	1.150	130	887	3.411
90	2.96	4.04	0012	1.013	1.095	087	925	3.382

$\nu = 2$ $\gamma = 1.3$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.398	0.408	0.424	0.449	0.489	0.550	0.646	0.792
10	382	389	402	426	466	532	640	805
15	372	379	395	422	464	537	652	812
20	367	376	394	427	475	554	669	817
0.25	0.367	0.376	0.392	0.429	0.492	0.571	0.681	0.827
30	369	375	397	438	500	591	697	832
35	376	382	406	454	520	609	719	845
40	387	395	422	471	542	629	737	861
45	400	403	441	491	560	647	749	869
0.50	0.420	0.431	0.464	0.516	0.580	0.665	0.763	0.874
55	448	461	494	542	606	684	778	883
60	488	500	528	571	633	705	793	892
65	543	548	568	603	659	727	808	900
70	612	606	614	640	685	749	822	907
0.75	0.687	0.676	0.669	0.681	0.716	0.771	0.838	0.915
80	760	750	736	732	752	796	856	924
85	825	822	814	800	794	823	872	933
90	885	885	883	878	866	861	893	945

$\nu = 2$ $\gamma = 1.3$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.0348	0.0547	0.0875	0.135	0.206	0.311	0.469	0.695
10	0230	0451	0827	141	231	368	570	812
15	[illegible]	0468	0998	180	283	424	614	827
20	0195	0554	130	234	348	481	638	810
0.25	0.0186	0.0665	0.165	0.283	0.410	0.540	0.679	0.834
30	0182	0849	210	341	466	591	714	848
35	0181	113	262	402	518	630	746	865
40	0183	152	325	459	565	664	770	882
45	0187	208	395	512	601	692	788	892
0.50	0.0192	0.285	0.464	0.558	0.633	0.716	0.804	0.898
55	0201	374	526	597	665	738	820	906
60	0215	466	580	631	694	759	834	915
65	0233	554	629	664	720	779	847	[illegible]
70	0247	633	676	699	745	799	860	928
0.75	0.0278	0.709	0.728	0.737	0.772	0.818	0.873	0.933
80	0300	779	787	780	803	838	887	941
85	0320	842	852	837	837	861	900	948
90	0338	892	908	900	896	891	916	958

$\nu = 2$ $\gamma = 1.3$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.474	0.514	0.552	0.591	0.632	0.676	0.722	0.773
10	389	435	478	521	565	613	666	723
15	318	366	416	461	503	556	616	676
20	259	307	359	406	449	505	568	632
0.25	0.212	0.260	0.302	0.350	0.407	0.458	0.519	0.587
30	161	209	256	301	353	415	474	539
35	126	168	208	257	309	370	434	498
40	091	152	167	212	269	328	393	458
45	059	092	126	169	223	285	349	415
0.50	0.033	0.056	0.086	0.129	0.180	0.242	0.306	0.370
55	0.012	[illegible]	051	092	143	202	266	328
60	-0.003	0.000	0.021	058	109	164	226	288
65	-0.011	-0.018	-0.004	027	074	128	186	246
70	-0.010	-0.027	-0.022	001	040	092	147	204
0.75	-0.006	-0.024	-0.031	-0.019	0.012	0.058	0.110	0.163
80	-0.002	-0.014	-0.026	-0.030	-0.010	0.027	074	124
85	-0.000	-0.005	-0.012	-0.023	-0.027	-0.001	039	084
90	000	[illegible]	-0.003	-0.006	-0.016	-0.020	007	048

$\nu = 2$ $\gamma = 1.3$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	11.4	7.45	4.84	3.32	2.38	1.770	1.377	1.139
10	16.6	8.63	4.86	3.01	2.02	1.448	1.121	0.992
15	17.9	8.11	3.96	2.34	1.639	1.267	1.061	0.982
20	18.9	6.78	3.03	1.829	1.367	1.152	1.049	1.008
0.25	19.7	5.65	2.38	1.517	1.200	1.055	1.003	0.992
30	20.3	4.42	1.893	1.285	1.072	0.999	0.977	981
35	20.7	3.38	1.549	1.129	1.002	966	964	977
40	21.1	2.61	1.297	1.025	0.960	947	957	976
45	21.4	1.969	1.116	0.959	932	935	951	974
0.50	21.8	1.511	1.001	0.924	0.916	0.928	0.949	0.973
55	22.2	1.232	0.938	909	911	927	950	974
60	22.7	1.072	910	906	912	929	952	976
65	23.3	0.988	902	909	915	933	954	977
70	24.0	955	907	915	920	937	957	978
0.75	24.7	0.952	0.919	0.924	0.927	0.943	0.961	0.980
80	25.3	963	936	938	937	949	965	982
85	25.8	977	955	956	948	956	969	984
90	26.2	991	973	976	968	966	974	987

$\nu = 2$ $\gamma = 1.3$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.237	0.251	0.268	0.296	0.340	0.413	0.530	0.715
10	256	266	281	308	355	436	570	764
15	273	280	297	328	374	457	590	779
20	287	296	317	351	402	488	616	784
0.25	0.301	0.312	0.329	0.366	0.433	0.516	0.636	0.800
30	316	324	345	386	451	545	659	808
35	332	340	362	410	477	570	686	825
40	351	359	385	434	506	595	709	844
45	371	379	410	460	528	617	725	854
0.50	0.395	0.406	0.439	0.489	0.553	0.639	0.742	0.862
55	428	441	472	520	583	663	760	872
60	472	483	511	553	614	687	779	884
65	530	534	554	589	644	713	796	893
70	602	596	603	629	674	738	813	902
0.75	0.680	0.668	0.662	0.674	0.708	0.763	0.832	0.910
80	754	745	732	727	747	790	851	922
85	822	819	810	797	791	820	869	931
90	884	883	882	877	865	859	891	944

$\nu = 2$ $\gamma = 1.4$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty} q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{u_n}{c}$	$\frac{T_n}{T_\infty} q$	$\frac{E_n}{p_\infty} q$
0.05	0.0097	0.0999	0.380	1.158	4.800	0.792	0.241	5.002
10	0207	148	292	1.150	4.000	750	288	4.450
15	0339	193	234	1.142	3.429	708	333	4.058
20	0498	238	194	1.133	3.000	667	378	3.767
0.25	0.0687	0.285	0.160	1.125	2.667	0.625	0.422	3.542
30	0901	333	132	1.117	2.400	583	465	3.363
35	116	387	110	1.108	2.182	542	508	3.219
40	148	448	0877	1.100	2.000	500	550	3.100
45	186	518	0698	1.092	1.846	458	592	3.001
0.50	0.234	0.599	0.0543	1.083	1.714	0.417	0.632	2.917
55	295	700	0409	1.075	1.600	375	672	2.845
60	376	826	0295	1.067	1.500	333	711	2.783
65	485	0.988	0202	1.058	1.412	292	749	2.730
70	637	1.21	0131	1.050	1.333	250	788	2.683
0.75	0.861	1.52	0.0081	1.042	1.263	0.208	0.825	2.643
80	1.20	1.98	0047	1.033	1.200	167	861	2.607
85	1.86	2.83	0022	1.025	1.143	125	897	2.575
90	3.16	4.48	0009	1.017	1.091	083	932	2.547

$\nu = 2$ $\gamma = 1.4$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.378	0.390	0.409	0.439	0.483	0.550	0.650	0.796
10	362	372	388	416	462	533	643	804
15	353	366	384	411	462	539	654	813
20	348	364	382	415	471	554	670	821
0.25	0.347	0.358	0.380	0.421	0.478	0.563	0.678	0.823
30	352	364	385	429	497	582	695	835
35	359	372	397	442	512	603	711	844
40	370	382	412	461	528	622	728	853
45	386	399	431	484	551	640	746	864
0.50	0.407	0.422	0.455	0.508	0.576	0.660	0.762	0.875
55	437	452	484	534	601	680	776	884
60	479	491	519	563	626	700	789	891
65	535	540	559	595	652	722	804	898
70	605	599	606	632	679	744	820	905
0.75	0.680	0.569	0.662	0.674	0.710	0.767	0.836	0.913
80	752	743	729	724	745	791	853	923
85	820	817	809	796	789	819	870	931
90	882	881	879	875	863	857	890	944

$\nu = 2$ $\gamma = 1.4$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.0466	0.0719	0.111	0.165	0.240	0.348	0.502	0.713
10	0351	0508	105	170	263	396	580	791
15	0287	0617	122	202	307	446	621	810
20	0258	0705	150	249	363	497	649	820
0.25	0.0245	0.0837	0.182	0.299	0.420	0.550	0.689	0.838
30	0240	103	224	352	478	598	723	857
35	0239	130	276	409	528	639	750	871
40	0241	169	338	446	570	673	776	883
45	0246	222	404	522	609	702	797	894
0.50	0.0254	0.294	0.479	0.568	0.646	0.726	0.814	0.906
55	0266	380	533	607	677	748	828	913
60	0283	473	589	641	705	769	841	919
65	0305	561	639	675	731	789	853	925
70	0333	643	687	710	755	808	866	931
0.75	0.0362	0.718	0.738	0.747	0.782	0.827	0.880	0.938
80	0388	785	793	788	810	845	893	944
85	0413	846	858	844	844	866	905	950
90	0435	896	911	903	900	896	920	960

$\nu = 2$ $\gamma = 1.4$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.307	0.375	0.435	0.492	0.548	0.606	0.666	0.728
10	308	387	433	478	525	576	631	690
15	274	343	386	427	477	530	588	649
20	222	276	322	367	420	477	540	605
0.25	0.170	0.219	0.270	0.322	0.371	0.423	0.493	0.559
30	134	185	226	276	334	389	452	518
35	101	149	189	233	289	350	410	476
40	071	112	150	194	246	309	371	434
45	046	078	113	158	208	270	333	395
0.50	0.026	0.048	0.078	0.121	0.173	0.231	0.294	0.356
55	0.008	0.021	046	086	138	193	255	316
60	-0.004	-0.001	0.018	054	104	157	216	276
65	-0.009	-0.018	-0.004	025	070	123	178	236
70	-0.008	-0.025	-0.021	0.001	039	089	141	196
0.75	-0.005	-0.022	-0.029	-0.018	0.012	0.056	0.106	0.156
80	-0.002	-0.013	-0.024	-0.028	-0.009	0.027	073	120
85	0.000	-0.005	-0.011	-0.021	-0.025	-0.001	038	081
90	000	-0.001	-0.003	-0.006	-0.015	-0.018	007	046

$\nu = 2$ $\gamma = 1.4$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	8.09	5.42	3.69	2.66	2.01	1.583	1.296	1.116
10	10.32	6.11	3.67	2.44	1.754	1.347	1.109	1.016
15	12.28	5.93	3.16	2.03	1.503	1.210	1.053	1.004
20	13.52	5.16	2.54	1.670	1.299	1.115	1.032	1.001
0.25	14.15	4.27	2.08	1.409	1.137	1.024	0.984	0.982
30	14.64	3.53	1.720	1.219	1.041	0.973	961	975
35	15.03	2.86	1.436	1.079	0.970	943	948	969
40	15.36	2.26	1.220	0.984	927	924	939	966
45	15.70	1.791	1.068	927	904	913	937	966
0.50	16.05	1.435	0.968	0.895	0.892	0.909	0.936	0.967
55	16.45	1.189	909	880	887	908	936	968
60	16.95	1.037	881	878	888	911	939	969
65	17.53	0.961	875	882	892	915	942	971
70	18.17	932	882	891	899	921	945	972
0.75	18.81	0.933	0.897	0.903	0.909	0.928	0.951	0.975
80	19.37	947	918	919	920	936	956	978
85	19.86	965	943	943	935	945	961	980
90	20.3	983	965	968	959	957	967	984

$\nu = 2$ $\gamma = 1.4$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.219	0.236	0.256	0.287	0.336	0.413	0.534	0.718
10	234	246	264	294	346	430	563	748
15	250	262	281	311	366	453	587	774
20	262	277	296	329	389	480	603	786
0.25	0.276	0.288	0.308	0.349	0.410	0.500	0.626	0.792
30	292	305	324	369	439	528	650	808
35	309	321	345	390	462	556	672	821
40	328	339	368	417	484	580	694	831
45	351	362	394	446	512	604	717	846
0.50	0.378	0.391	0.424	0.476	0.543	0.628	0.735	0.860
55	413	427	458	506	572	653	754	871
60	458	471	498	540	603	678	771	880
65	519	523	542	577	633	704	789	889
70	592	596	593	618	664	730	808	898
0.75	0.670	0.650	0.653	0.664	0.700	0.756	0.827	0.908
80	745	736	723	718	739	784	847	919
85	816	813	805	792	785	815	866	928
90	880	880	878	873	861	856	888	943

$\nu = 2$ $\gamma = 5/3$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty} q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n}{c}$	$\frac{T_n}{T_\infty} q$	$\frac{E_n}{p_\infty} q$
0.05	0.0107	0.121	0.200	1.238	3.478	0.712	0.356	3.328
10	0229	178	140	1.225	3.077	675	398	3.006
15	0375	232	106	1.213	2.759	638	440	2.753
20	0546	286	0856	1.200	2.500	600	480	2.550
0.25	0.0745	0.340	0.0691	1.188	2.286	0.562	0.520	2.384
30	0986	399	0556	1.175	2.105	525	559	2.246
35	128	464	0445	1.163	1.951	488	596	2.130
40	162	538	0354	1.150	1.818	450	633	2.032
45	204	621	0278	1.137	1.702	412	668	1.948
0.50	0.257	0.720	0.0214	1.125	1.600	0.375	0.703	1.875
55	324	840	0159	1.112	1.509	338	737	1.812
60	412	0.989	0115	1.100	1.429	300	770	1.757
65	531	1.18	0079	1.087	1.356	262	802	1.709
70	698	1.44	0052	1.075	1.290	225	833	1.667
0.75	0.946	1.82	0.0032	1.062	1.231	0.188	0.863	1.630
80	1.31	2.36	0019	1.050	1.176	150	893	1.597
85	2.05	3.41	0009	1.037	1.127	112	920	1.568
90	3.32	5.15	0004	1.025	1.081	075	948	1.543

$\nu = 2$ $\gamma = 5/3$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.324	0.336	0.357	0.391	0.442	0.518	0.628	0.785
10	315	325	344	378	431	511	628	792
15	311	322	343	378	433	521	641	800
20	311	324	347	386	445	538	659	810
0.25	0.316	0.326	0.350	0.395	0.460	0.551	0.675	0.825
30	322	332	357	404	475	568	687	834
35	330	342	370	419	492	586	702	841
40	343	356	387	438	512	607	719	850
45	361	374	408	461	534	628	737	860
0.50	0.385	0.399	0.434	0.487	0.558	0.648	0.753	0.870
55	418	431	464	515	584	669	769	880
60	462	472	499	545	611	690	784	888
65	520	522	540	578	638	712	799	896
70	590	583	589	646	666	734	814	904
0.75	0.665	0.655	0.647	0.658	0.697	0.757	0.830	0.911
80	737	728	715	710	731	781	846	920
85	810	806	798	785	778	809	863	928
90	869	868	866	860	848	845	882	940

$\nu = 2$ $\gamma = 5/3$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.0434	0.0806	0.130	0.193	0.277	0.389	0.540	0.740
10	0309	0737	131	206	304	433	599	788
15	0261	0782	150	239	347	482	642	814
20	0252	0903	180	284	398	531	677	832
0.25	0.0241	0.106	0.213	0.330	0.451	0.579	0.715	0.856
30	0236	128	256	382	505	623	746	873
35	0235	159	308	438	554	664	773	885
40	0236	200	367	494	598	698	796	896
45	0241	255	431	546	638	726	816	907
0.50	0.0248	0.325	0.497	0.593	0.672	0.751	0.832	0.915
55	0259	407	559	634	703	773	848	923
60	0274	496	615	669	731	792	859	929
65	0294	584	665	702	756	811	872	935
70	0316	665	713	735	780	828	882	940
0.75	0.0339	0.737	0.762	0.769	0.803	0.845	0.893	0.945
80	0360	800	813	807	828	861	904	951
85	.0381	858	871	859	860	880	916	956
90	0398	900	916	908	905	903	927	963

$\nu = 2$ $\gamma = 5/3$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.250	0.310	0.364	0.418	0.472	0.529	0.588	0.650
10	193	259	315	370	426	484	546	610
15	152	218	276	329	384	446	509	572
20	121	184	241	293	347	410	474	536
0.25	0.092	0.154	0.206	0.259	0.314	0.373	0.438	0.501
30	071	129	174	224	280	338	400	464
35	052	104	144	190	245	304	364	426
40	036	079	116	159	212	270	330	390
45	023	055	088	130	180	238	296	354
0.50	0.011	0.032	0.061	0.101	0.149	0.205	0.263	0.319
55	0.003	0.013	036	073	120	173	229	284
60	-0.003	-0.004	0.013	046	092	141	195	248
65	-0.005	-0.015	-0.005	022	064	111	162	213
70	-0.004	-0.020	-0.019	001	036	081	129	177
0.75	-0.002	-0.018	-0.024	-0.016	0.012	0.052	0.096	0.142
80	-0.001	-0.011	-0.020	-0.024	-0.008	026	067	109
85	0.000	-0.004	-0.010	-0.018	-0.022	0.000	035	073
90	000	-0.002	-0.003	-0.006	-0.014	-0.016	009	044

$\nu = 2$ $\gamma = 5/3$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	7.46	4.17	2.76	2.02	1.598	1.333	1.163	1.061
10	10.19	4.41	2.63	1.834	1.416	1.179	1.049	1.005
15	11.92	4.12	2.28	1.581	1.250	1.082	0.999	0.982
20	12.36	3.58	1.927	1.360	1.119	1.013	0.974	974
0.25	13.08	3.08	1.641	1.196	1.018	0.952	0.944	0.954
30	13.62	2.59	1.397	1.058	0.942	910	921	955
35	14.09	2.15	1.201	0.956	889	884	908	950
40	14.54	1.7[illegible]7	1.052	887	856	870	904	949
45	15.00	1.468	0.946	844	838	864	903	949
0.50	15.52	1.230	0.873	0.821	0.831	0.863	0.905	0.951
55	16.12	1.059	830	813	831	865	908	953
60	16.85	0.951	812	815	836	870	912	956
65	17.71	895	812	824	844	878	917	958
70	18.66	878	826	838	854	886	922	961
0.75	19.60	0.888	0.849	0.856	0.868	0.896	0.929	0.964
80	20.4	910	879	879	883	906	936	967
85	21.2	939	916	914	905	919	943	970
90	21.8	964	945	947	936	935	951	976

$\nu = 2$ $\gamma = 5/3$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.180	0.194	0.214	0.248	0.301	0.384	0.513	0.708
10	193	203	222	256	312	401	537	727
15	206	216	236	272	329	424	563	751
20	220	232	253	291	353	453	589	766
0.25	0.236	0.247	0.268	0.312	0.379	0.476	0.614	0.788
30	252	263	285	332	404	500	632	801
35	271	280	306	355	428	527	653	811
40	292	301	331	381	454	553	675	823
45	316	327	360	411	483	580	697	836
0.50	0.347	0.359	0.392	0.443	0.513	0.606	0.719	0.850
55	385	397	428	477	545	632	739	862
60	434	443	469	513	578	658	758	873
65	496	499	516	532	611	686	778	884
70	570	564	570	595	645	714	797	893
0.75	0.650	0.640	0.633	0.644	0.682	0.742	0.817	0.902
80	726	718	705	700	721	770	837	914
85	804	800	792	779	772	803	858	924
90	867	865	863	853	845	842	880	938

$\nu = 3$ $\gamma = 1.3$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty}q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n}{c}$	$\frac{T_n}{T_\infty}q$	$\frac{E_n}{p_\infty}q$
0.05	0.0146	0.181	0.470	1.124	5.750	0.826	0.196	6.297
10	0274	234	378	1.117	4.600	783	242	5.556
15	0415	281	313	1.110	3.833	739	290	5.064
20	0569	323	264	1.104	3.286	696	336	4.715
0.25	0.0737	0.363	0.221	1.098	2.875	0.632	0.382	4.454
30	0918	402	186	1.091	2.556	609	427	4.253
35	113	444	156	1.085	2.300	565	472	4.094
40	137	489	128	1.078	2.091	522	516	3.964
45	164	536	104	1.072	1.917	478	559	3.857
0.50	0.196	0.590	0.0832	1.065	1.769	0.435	0.602	3.768
55	236	653	0639	1.059	1.643	391	644	3.692
60	287	729	0740	1.052	1.533	348	686	3.628
65	349	819	0329	1.046	1.438	304	727	3.572
70	429	0.930	0217	1.039	1.353	261	768	3.524
0.75	0.541	1.08	0.0132	1.033	1.278	0.217	0.808	3.481
80	713	1.30	0072	1.026	1.211	174	848	3.444
85	0.980	1.64	0035	1.020	1.150	130	887	3.411
90	1.41	2.16	0015	1.013	1.095	087	925	3.382

$\nu = 3$ $\gamma = 1.3$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.402	0.415	0.435	0.465	0.509	0.574	0.670	0.810
10	383	394	411	440	485	556	666	825
15	370	385	403	431	482	562	676	832
20	362	380	402	435	491	578	692	836
0.25	0.360	0.370	0.396	0.444	0.504	0.587	0.702	0.840
30	364	375	399	450	527	610	719	852
35	367	380	410	462	541	635	736	859
40	374	387	425	483	556	655	756	868
45	385	400	444	507	580	675	777	882
0.50	0.400	0.418	0.466	0.533	0.606	0.694	0.793	0.894
55	419	443	493	559	633	713	806	903
60	448	476	526	587	662	734	819	909
65	492	516	564	618	693	757	834	917
70	558	564	606	653	721	783	850	926
0.75	0.642	0.625	0.653	0.694	0.744	0.808	0.866	0.933
80	734	703	705	739	772	832	884	938
85	818	792	771	787	816	855	904	947
90	888	876	854	843	864	889	925	965

$\nu = 3$ $\gamma = 1.3$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.0460	0.0657	0.101	0.153	0.227	0.337	0.497	0.715
10	0311	0548	096	158	253	395	600	827
15	0259	0550	113	193	301	448	639	843
20	0252	0628	142	244	361	501	658	828
0.25	0.0241	0.0746	0.174	0.296	0.423	0.556	0.698	0.844
30	0236	0929	214	348	486	607	730	863
35	0234	118	266	407	535	650	759	876
40	0234	153	327	467	576	685	786	888
45	0237	202	391	521	616	714	810	902
0.50	0.0243	0.267	0.455	0.568	0.653	0.739	0.828	0.914
55	0251	348	517	608	686	761	842	922
60	0264	435	573	643	718	782	855	928
65	0283	518	623	676	748	804	868	934
70	0311	593	669	709	774	826	881	942
0.75	0.0346	0.665	0.714	0.747	0.795	0.848	0.895	0.948
80	0384	740	761	786	819	867	909	952
85	0418	817	817	826	855	886	925	959
90	0444	886	885	872	893	913	942	973

$\nu = 3$ $\gamma = 1.3$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.562	0.591	0.619	0.649	0.680	0.712	0.748	0.786
10	483	516	548	581	616	658	694	737
15	416	455	488	517	554	601	644	693
20	354	395	430	459	497	546	597	649
0.25	0.283	0.320	0.366	0.414	0.451	0.495	0.548	0.600
30	240	281	314	359	412	453	503	558
35	193	236	270	308	361	412	459	512
40	149	188	225	265	311	370	419	468
45	108	143	180	225	270	327	381	428
0.50	0.074	0.100	0.137	0.184	0.232	0.285	0.340	0.389
55	041	061	096	144	193	243	297	347
60	0.008	0.025	058	104	158	203	255	303
65	-0.019	-0.008	027	069	125	166	214	261
70	-0.030	-0.033	0.001	038	090	133	176	221
0.75	-0.023	-0.047	-0.022	0.013	0.053	0.100	0.138	0.179
80	-0.013	-0.046	-0.039	-0.008	019	065	101	136
85	-0.005	-0.029	-0.041	-0.025	0.001	032	067	096
90	-0.001	-0.009	-0.025	-0.031	-0.012	008	036	064

$\nu = 3$ $\gamma = 1.3$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	8.73	6.31	4.29	3.04	2.24	1.706	1.350	1.134
10	12.32	7.18	4.30	2.78	1.919	1.409	1.110	0.997
15	14.27	7.00	3.57	2.23	1.601	1.254	1.058	0.987
20	14.33	6.06	2.82	1.782	1.359	1.153	1.052	1.010
0.25	14.54	4.96	2.28	1.499	1.191	1.056	1.006	0.994
30	15.38	4.03	1.863	1.291	1.086	1.005	0.984	987
35	15.69	3.22	1.539	1.135	1.011	0.976	969	980
40	15.95	2.53	1.301	1.033	0.967	956	962	977
45	16.19	1.984	1.135	0.973	942	945	959	978
0.50	16.43	1.567	1.024	0.938	0.928	0.939	0.958	0.979
55	16.67	1.274	0.953	919	922	937	958	979
60	16.97	1.093	917	913	922	938	958	980
65	17.38	0.995	905	914	926	942	961	981
70	17.91	951	906	920	931	947	964	983
0.75	18.51	0.940	0.914	0.929	0.936	0.953	0.968	0.984
80	19.10	950	926	940	943	959	972	985
85	19.60	969	944	952	954	965	977	988
90	19.97	989	966	967	967	973	982	992

$\nu = 3$ $\gamma = 1.3$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.239	0.261	0.282	0.315	0.366	0.444	0.565	0.742
10	257	272	292	326	380	469	607	789
15	272	287	308	340	397	491	625	807
20	282	302	328	362	423	519	646	810
0.25	0.296	0.310	0.336	0.386	0.450	0.539	0.665	0.817
30	311	326	349	401	483	571	687	834
35	324	339	369	421	503	601	708	843
40	339	353	390	449	524	626	733	853
45	356	371	414	479	552	650	757	870
0.50	0.376	0.394	0.442	0.508	0.582	0.672	0.777	0.885
55	400	423	473	538	612	695	792	895
60	433	460	509	569	645	718	807	902
65	480	503	550	603	679	744	824	911
70	548	555	596	642	710	773	842	921
0.75	0.634	0.618	0.645	0.686	0.737	0.802	0.860	0.930
80	728	698	701	734	767	827	880	935
85	815	789	769	784	813	852	901	945
90	886	875	853	842	862	887	924	964

$\nu = 3$ $\gamma = 1.4$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty}q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n}{c}$	$\frac{T_n}{T_\infty}q$	$\frac{E_n}{p_\infty}q$
0.05	0.0151	0.194	0.350	1.158	4.800	0.792	0.241	5.002
10	0284	252	270	1.150	4.000	750	288	4.450
15	0425	300	218	1.142	3.429	708	333	4.058
20	0580	344	181	1.133	3.000	667	378	3.767
0.25	0.0755	0.388	0.151	1.125	2.667	0.625	0.422	3.542
30	0950	432	125	1.117	2.400	583	465	3.363
35	116	477	104	1.108	2.182	542	508	3.219
40	141	524	0848	1.100	2.000	500	550	3.100
45	170	576	0682	1.092	1.846	458	592	3.001
0.50	0.204	0.635	0.0538	1.083	1.714	0.417	0.632	2.917
55	245	702	0412	1.075	1.600	375	672	2.845
60	.296	781	0305	1.067	1.500	333	711	2.783
65	359	876	0210	1.058	1.412	292	749	2.730
70	441	0.994	0138	1.050	1.333	250	788	2.683
0.75	0.557	1.16	0.0084	1.042	1.263	0.208	0.825	2.643
80	0.735	1.39	0048	1.033	1.200	167	861	2.607
85	1.02	1.76	0024	1.025	1.143	125	897	2.575
90	1.47	2.33	0008	1.017	1.091	083	932	2.547

$\nu = 3$ $\gamma = 1.4$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.371	0.384	0.404	0.435	0.481	0.550	0.650	0.797
10	357	367	385	415	463	537	649	809
15	348	362	380	413	467	549	668	822
20	343	360	381	419	480	568	690	833
0.25	0.339	0.355	0.331	0.423	0.488	0.579	0.696	0.840
30	340	353	382	434	503	596	710	846
35	346	360	391	448	526	618	730	857
40	355	370	405	466	549	642	748	869
45	366	383	426	489	571	665	766	878
0.50	0.381	0.402	0.450	0.516	0.594	0.687	0.783	0.887
55	402	428	479	546	621	706	801	896
60	433	462	514	577	652	730	818	906
65	480	503	553	609	685	754	835	917
70	547	553	596	645	716	779	850	927
0.75	0.632	0.615	0.644	0.686	0.740	0.805	0.865	0.934
80	722	698	692	733	775	822	887	939
85	814	786	766	781	812	852	901	946
90	884	873	850	839	860	886	923	964

$\nu = 3$ $\gamma = 1.4$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.0361	0.0593	0.0957	0.148	0.222	0.330	0.488	0.706
10	0249	0509	0928	155	248	382	572	792
15	0208	0526	107	188	294	438	622	819
20	0200	0602	133	234	353	496	657	825
0.25	0.0189	0.0696	0.166	0.283	0.414	0.552	0.698	0.849
30	0184	0866	205	340	473	603	733	865
35	0182	110	254	400	531	648	764	880
40	0183	143	314	460	580	686	789	894
45	0185	191	381	518	622	718	811	905
0.50	0.0190	0.256	0.452	0.570	0.658	0.747	0.830	0.914
55	0196	338	518	613	692	771	848	922
60	0206	428	578	651	725	792	863	931
65	0222	516	630	685	756	813	877	939
70	0243	596	678	713	784	834	890	946
0.75	0.0269	0.671	0.723	0.756	0.804	0.855	0.901	0.952
80	0296	742	773	794	834	874	923	942
85	0322	823	824	832	861	892	928	961
90	0341	890	889	876	898	917	945	974

$\nu = 3$ $\gamma = 1.4$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.473	0.510	0.547	0.583	0.620	0.660	0.702	0.746
10	401	444	484	523	563	606	652	701
15	345	389	429	470	514	560	611	660
20	294	340	380	422	467	517	571	620
0.25	0.237	0.287	0.333	0.374	0.419	0.468	0.522	0.576
30	190	237	282	329	374	425	478	531
35	152	200	238	286	336	385	438	490
40	117	163	199	244	297	347	399	451
45	085	124	160	204	256	309	359	410
0.50	0.055	0.086	0.122	0.167	0.216	0.271	0.321	0.369
55	029	051	086	132	180	233	283	329
60	0.002	0.019	053	098	149	196	246	290
65	-0.018	-0.010	025	066	119	161	208	252
70	-0.026	-0.032	0.000	037	088	128	171	213
0.75	-0.021	-0.045	-0.021	0.013	0.052	0.097	0.134	0.174
80	-0.012	-0.036	-0.029	-0.003	028	062	100	136
85	-0.004	-0.027	-0.039	-0.024	0.001	031	065	092
90	-0.001	-0.009	-0.024	-0.030	-0.011	008	035	062

$\nu = 3$ $\gamma = 1.4$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	10.28	6.47	4.22	2.95	2.16	1.662	1.333	1.128
10	14.34	7.21	4.15	2.67	1.871	1.404	1.134	1.022
15	16.74	6.88	3.55	2.20	1.587	1.252	1.074	1.004
20	17.18	5.99	2.87	1.789	1.360	1.147	1.050	1.009
0.25	17.93	5.10	2.30	1.495	1.181	1.048	0.998	0.989
30	18.51	4.08	1.865	1.274	1.062	0.987	968	978
35	18.98	3.27	1.539	1.120	0.991	953	956	974
40	19.38	2.59	1.295	1.014	946	936	949	972
45	19.73	2.01	1.117	0.945	918	925	944	971
0.50	20.1	1.569	0.996	0.907	0.903	0.920	0.944	0.971
55	20.5	1.268	925	890	898	920	946	972
60	21.0	1.080	889	886	900	922	948	974
65	21.7	0.976	877	889	906	927	952	976
70	22.5	928	880	897	913	934	956	979
0.75	23.5	0.918	0.891	0.909	0.920	0.941	0.960	0.981
80	24.4	932	907	923	927	948	964	981
85	25.3	955	928	938	942	955	971	984
90	25.9	981	956	957	958	966	978	990

$\nu = 3$ $\gamma = 1.4$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.215	0.233	0.253	0.286	0.337	0.416	0.538	0.723
10	231	244	263	297	351	439	575	760
15	247	259	279	315	375	468	607	789
20	258	275	298	338	402	502	638	803
0.25	0.269	0.287	0.312	0.355	0.426	0.523	0.653	0.816
30	282	298	324	378	450	549	673	824
35	298	313	342	401	481	578	698	838
40	315	331	365	426	511	607	720	853
45	332	349	391	454	537	634	741	864
0.50	0.353	0.373	0.420	0.486	0.564	0.660	0.762	0.874
55	379	404	454	520	595	685	783	885
60	415	443	493	555	631	712	803	898
65	466	488	536	592	668	738	823	910
70	535	542	583	631	702	767	841	922
0.75	0.623	0.607	0.634	0.676	0.730	0.796	0.858	0.930
80	712	681	683	716	747	811	863	919
85	810	782	762	778	807	848	898	944
90	882	871	848	837	858	884	922	964

$\nu = 3$ $\gamma = 5/3$

q	τ	R_n	ξ_0	$\frac{p_n}{p_\infty} q$	$\frac{\rho_n}{\rho_\infty}$	$\frac{v_n'}{c}$	$\frac{T_n}{T_\infty} q$	$\frac{E_n}{p_\infty} q$
0.05	0.0156	0.220	0.350	1.238	3.478	0.712	0.356	3.328
10	0293	285	260	1.225	3.077	675	398	3.006
15	0441	340	202	1.213	2.759	638	440	2.753
20	0600	389	163	1.200	2.500	600	480	2.550
0.25	0.0772	0.436	0.132	1.188	2.286	0.562	0.520	2.384
30	0978	487	105	1.175	2.105	525	559	2.246
35	121	540	0843	1.163	1.951	488	596	2.130
40	148	595	0675	1.150	1.818	450	633	2.032
45	178	655	0535	1.137	1.702	412	668	1.948
0.50	0.213	0.722	0.0415	1.125	1.600	0.375	0.703	1.875
55	257	799	0313	1.112	1.509	338	737	1.812
60	310	889	0228	1.100	1.429	300	770	1.757
65	376	0.996	0159	1.087	1.356	262	802	1.709
70	461	1.13	0105	1.075	1.290	225	833	1.667
0.75	0.580	1.31	0.0065	1.062	1.231	0.188	0.863	1.630
80	0.763	1.58	0036	1.050	1.176	150	893	1.597
85	1.05	1.99	0017	1.037	1.127	112	920	1.568
90	1.52	2.63	0007	1.025	1.081	075	948	1.543

$\nu = 3$ $\gamma = 5/3$

q	$\frac{p_0}{p_n}$	$\frac{p_1}{p_n}$	$\frac{p_2}{p_n}$	$\frac{p_3}{p_n}$	$\frac{p_4}{p_n}$	$\frac{p_5}{p_n}$	$\frac{p_6}{p_n}$	$\frac{p_7}{p_n}$
0.05	0.341	0.366	0.401	0.448	0.510	0.592	0.697	0.832
10	319	341	374	420	485	573	688	833
15	296	328	366	420	490	585	697	833
20	281	324	365	427	505	606	716	840
0.25	0.285	0.324	0.360	0.422	0.509	0.612	0.732	0.861
30	289	324	363	422	510	616	735	866
35	295	329	371	434	517	625	742	867
40	305	340	386	453	537	640	755	874
45	319	356	407	476	560	658	768	883
0.50	0.338	0.376	0.431	0.502	0.584	0.678	0.782	0.890
55	364	403	460	530	612	699	796	897
60	402	437	494	561	642	722	812	905
65	456	480	533	594	673	746	826	914
70	528	532	576	631	703	772	845	923
0.75	0.616	0.596	0.624	0.673	0.729	0.798	0.861	0.931
80	712	678	680	719	758	821	878	936
85	802	773	751	768	803	846	898	944
90	877	864	840	828	852	881	920	963

$\nu = 3$ $\gamma = 5/3$

q	$\frac{\rho_0}{\rho_n}$	$\frac{\rho_1}{\rho_n}$	$\frac{\rho_2}{\rho_n}$	$\frac{\rho_3}{\rho_n}$	$\frac{\rho_4}{\rho_n}$	$\frac{\rho_5}{\rho_n}$	$\frac{\rho_6}{\rho_n}$	$\frac{\rho_7}{\rho_n}$
0.05	0.108	0.143	0.199	0.270	0.359	0.473	0.616	0.788
10	0789	124	187	269	372	503	659	820
15	0636	121	193	292	406	543	692	838
20	0573	128	218	327	450	586	721	854
0.25	0.0549	0.140	0.244	0.361	0.492	0.624	0.755	0.880
30	0536	159	280	406	534	660	780	895
35	0530	186	327	457	576	694	802	903
40	0532	222	380	509	619	723	820	911
45	0540	268	437	559	658	748	837	921
0.50	0.0555	0.326	0.497	0.605	0.691	0.772	0.851	0.928
55	0578	397	556	645	722	794	865	934
60	0610	474	609	680	752	814	878	939
65	0656	551	657	713	780	834	890	946
70	0715	624	702	746	805	854	902	953
0.75	0.0783	0.694	0.744	0.779	0.824	0.872	0.913	0.958
80	0855	767	789	814	846	889	925	961
85	0917	838	840	848	876	904	937	966
90	0968	899	900	888	908	927	951	978

$\nu = 3$ $\gamma = 5/3$

q	$\frac{v_0}{c}$	$\frac{v_1}{c}$	$\frac{v_2}{c}$	$\frac{v_3}{c}$	$\frac{v_4}{c}$	$\frac{v_5}{c}$	$\frac{v_6}{c}$	$\frac{v_7}{c}$
0.05	0.407	0.445	0.482	0.519	0.557	0.595	0.634	0.674
10	337	380	421	462	504	546	590	633
15	276	321	372	421	466	512	552	593
20	231	271	327	382	431	479	519	556
0.25	0.180	0.232	0.278	0.332	0.388	0.438	0.484	0.525
30	144	194	237	284	340	393	441	486
35	113	158	200	245	295	351	400	446
40	083	126	166	210	260	312	363	408
45	057	094	134	178	226	277	326	372
0.50	0.032	0.063	0.102	0.146	0.194	0.242	0.290	0.335
55	0.009	034	072	116	163	209	254	298
60	-0.009	0.008	044	087	134	177	220	262
65	-0.021	-0.015	0.020	059	107	146	187	226
70	-0.023	-0.033	-0.002	015	079	117	154	192
0.75	-0.017	-0.042	-0.020	0.013	0.048	0.089	0.122	0.156
80	-0.009	-0.039	-0.034	-0.006	020	059	089	119
85	-0.003	-0.024	-0.036	-0.021	0.003	030	060	084
90	0.000	-0.008	-0.021	-0.027	-0.010	009	032	056

$\nu = 3$ $\gamma = 5/3$

q	$\frac{T_0}{T_n}$	$\frac{T_1}{T_n}$	$\frac{T_2}{T_n}$	$\frac{T_3}{T_n}$	$\frac{T_4}{T_n}$	$\frac{T_5}{T_n}$	$\frac{T_6}{T_n}$	$\frac{T_7}{T_n}$
0.05	3.15	2.56	2.02	1.662	1.420	1.250	1.132	1.056
10	4.04	2.75	1.995	1.566	1.302	1.138	1.043	1.016
15	4.65	2.72	1.860	1.437	1.207	1.077	1.008	0.995
20	4.91	2.54	1.673	1.304	1.123	1.034	0.992	0.983
0.25	5.19	2.31	1.479	1.168	1.035	0.980	0.970	0.978
30	5.40	2.04	1.294	1.041	0.953	933	942	967
35	5.57	1.772	1.136	0.949	898	901	926	961
40	5.73	1.537	1.018	889	867	885	920	959
45	5.90	1.328	0.930	851	852	880	918	959
0.50	6.09	1.152	0.867	0.830	0.846	0.879	0.918	0.960
55	6.31	1.015	828	822	847	881	920	960
60	6.59	0.923	812	825	853	887	925	963
65	6.95	871	811	834	863	895	930	966
70	7.38	852	821	847	874	904	936	969
0.75	7.86	0.859	0.839	0.864	0.884	0.915	0.943	0.972
80	8.34	885	862	884	896	925	950	974
85	8.75	923	894	906	916	935	958	977
90	9.07	961	934	933	938	950	967	985

$\nu = 3$ $\gamma = 5/3$

q	$\frac{E_0}{E_n}$	$\frac{E_1}{E_n}$	$\frac{E_2}{E_n}$	$\frac{E_3}{E_n}$	$\frac{E_4}{E_n}$	$\frac{E_5}{E_n}$	$\frac{E_6}{E_n}$	$\frac{E_7}{E_n}$
0.05	0.190	0.229	0.264	0.314	0.382	0.476	0.604	0.773
10	195	224	258	307	379	480	615	778
15	199	227	264	321	398	505	637	797
20	199	239	276	338	425	538	664	808
0.25	0.213	0.250	0.283	0.345	0.439	0.553	0.688	0.837
30	227	258	295	357	448	563	696	845
35	242	272	313	377	462	576	706	847
40	259	290	335	401	487	596	722	855
45	279	312	362	430	515	619	738	866
0.50	0.304	0.338	0.392	0.451	0.544	0.643	0.754	0.875
55	336	371	426	494	576	668	772	883
60	378	411	465	530	612	695	791	893
65	435	458	509	569	648	724	811	904
70	511	515	558	611	683	754	831	915
0.75	0.602	0.584	0.610	0.658	[illegible]	0.785	0.850	0.925
80	703	669	671	710	746	812	870	931
85	797	767	746	762	797	839	893	941
90	874	861	837	825	848	878	917	962